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DIVERGENT EXPECTATIONS, R&D EXPENDITURES AND TECHNICAL PROGRESS

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ABSTRACT

This paper deals with the problem of whether diversity of beliefs or consensus leads to a higher level of R and D spending in an industry, or a faster rate of technical progress, or a lower price for the industry's product. The answer depends upon characteristics of the demand function for industry output as well as the density functions reflecting probability beliefs of firm managers. Sufficient conditions are given for consensus (diversity) to ''pay'' in terms of R and D spending and/or the rate of technological progress, and R and D spending under market incentives is contrasted with an ''optimal'' level of R and D expenditures. The analysis is illustrated in more detail for the case of exponential density functions.

DIVERGENT EXPECTATIONS, R and D EXPENDITURES AND TECHNICAL PROGRESS[1]

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I. INTRODUCTION

This paper is concerned with the role that expectations play in the allocation of funds for research and development in an industry. In particular, we are interested in the question as to whether differences of beliefs among firm managers in an industry as to the prospective payoffs from R and D tend to increase or to decrease the amount of R and D spending within an industry, and what effect this has on the rate of technical progress in the industry. For example, when each firm manager believes that other firms have as good a chance of producing a profitable innovation as his firm, do differences of beliefs among firm managers as to the chances of success encourage or discourage R and D expenditures in the industry as compared to commonly shared beliefs as to the chances of success? To put it another way, does consensus or diversity ''pay'' in the sense of increasing the level of R and D expenditures in an industry? Viewed in this way, the problem considered in this paper is related to

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that of assessing the effects of government long term forecasts as to future prospects for an industry (assuming they are believed by firms in the industry) on R and D spending and on the rate of technical progress in the industry.

As a matter of definition, we describe an industry as operating under strong uncertainty if individuals who are knowledgeable about the industry (including firm managers) have different beliefs as to the future prospects for the industry. We take the view that individual participants in an industry all have subjective probability distributions over possible outcomes for the industry, but under strong uncertainty there simply haven't been enough public experiments conducted to enable the subjective probabilities of informed individuals to converge to a state of identical probabilities, as in the famous Savage (4) theorem.

So, in an industry operating under strong uncertainty, individuals make decisions concerning the future based on divergent subjective beliefs as to the occurrence of various states of the world. Mutual interactions among these decisions lead to a market equilibrium for the industry that involves a certain pattern of expenditures on the part of firms (including expenditures on R and D), and a certain pattern of market shares. Associated with this is a certain rate of technical progress. In principle at least, we can contrast such an equilibrium with one that obtains under weak uncertainty, defined as a situation in which informed individuals within an industry all have the same subjective probability beliefs

concerning future prospects for the industry. For example, if a government bureau announces a set of probabilities for possible future states of the world, and if firms in an industry accept these probabilities and act on the basis of them, then the industry would be operating under weak uncertainty.

In our analysis of the impact of diversity of beliefs or consensus on R and D activity in an industry, we employ a highly simplified model of the R and D process. In this model, firms can differ from one another in their beliefs as to the ''effectiveness'' of their R and D operations (the payoffs from R and D expenditures), and in the amounts spent on R and D. With consensus, beliefs to R and D effectiveness as well as R and D expenditures are the same for all firms; under divergent beliefs, they differ. In comparing the results obtained under divergent beliefs with those that hold under commonly shared beliefs, we assume that the commonly shared beliefs are centered at the mean of the distribution of the divergent beliefs. We examine the questions as to whether diversity or consensus leads to a higher level of R and D spending in an industry, and as to which is associated with a higher rate of technical progress.

Even with the simplifications introduced into our model of the R and D process, the results are ambiguous. Whether diversity or consensus will lead to larger R and D expenditures or to a faster rate of technical progress or to a lower price for the product depends in general on the probability distributions over the payoffs from R and D and on the properties of the demand curve for the industry's product.

However, sufficient conditions are stated such that diversity ''pays'' or consensus ''pays.'' In one special case -- that of exponential probability distributions -- these problems are examined in detail. Finally, some tentative statements are made concerning the optimality of a market oriented R and D process.

II. A MODEL OF R and D EXPENDITURES AND SUBJECTIVE BELIEFS

Consider an industry in which there are n firms, each engaged in R and D activities aimed at producing a standardized product. The number of firms in the industry is fixed, so that entry of new firms is not taken into account in our story of the industry. We do not attempt to model in detail the complex and interrelated decisions that are made by firms in the industry. Instead, we will examine a highly simplified and stylized R and D game that operates according to the following rules.

Firm i, i = 1, ..., n, invests an amount R_i today in R and D with the objective of producing units of the standardized product tomorrow. Let C_i denote the per unit cost of production of the product for firm i tomorrow, after R_i is spent today on R and D. C_i is taken to be a random variable with R_i a parameter of the probability density function over C_i. We assume constant returns so that any level of output can be produced tomorrow at the per unit cost C_i. Let $f_{ij}(C_j;R_j)$ denote the i^{th} firm's subjective probability over firm j's per unit cost tomorrow. Under weak uncertainty (commonly shared beliefs) there is agreement among all firms as to the

probability density function for any C_j; that is, under weak uncertainty we would have $f_{ij}(C_j;R_j) = f_{kj}(C_j;R_j)$ for every i, k = 1, ..., n and for every j. With strong uncertainty, there is no guarantee that beliefs of all firms concerning any one firm's prospects are identical.

We adopt the realistic assumption that at the time that firm i (i = 1, ..., n) must make its R and D investment decision, it does not know the amounts invested by other firms. After all R and D decisions have been taken and after the random variables are observed, then all of the per unit costs C_i(i = 1, ..., n) are made known. We restrict our attention to a simple two-period model in which there is R and D investment today and sale of the product tomorrow. Admittedly, there are interesting and important extension of this case to the multi-period horizon case, but even the two-period model is not without some analytical complications, and we have not as yet attempted to generalize our simplified structure.

To keep things manageable, we assume that all firms are expected profit maximizers. Then the decision problem for firm k is to choose R and D expenditures R_k today to maximize discounted expected profits, which are determined as follows.

Given the inverse industry demand function p = p(Q), and given a minimum per unit cost C_L among the firms in the industry, then the unconstrained monopoly price and output are such that $MC(Q^*) = C_L$, where Q^* is the monopoly output. Let $p^*(C_L)$ denote the monopoly price given a per unit cost of C_L, so that $Q^* \frac{dp^*(C_L)}{dQ} + p^*(C_L) = C_L$.

Consider expected profits E_k for firm k. Firm k spends R_k dollars this period on R and D. Next period, firm k earns nothing unless it is the low cost firm. If firm k is the low cost firm, then it becomes the sole seller of the product. It will charge what the market will bear. This is the unconstrained monopoly price p^* if it is free to do so, that is, if the second lowest cost firm has a per unit cost in excess of p^*. However, if the per unit cost of the second lowest cost firm is less than the monopoly price p^*, firm k charges a price equal to the per unit cost of the second lowest cost firm.

Let C_S denote the per unit cost of the second lowest cost firm with C_L the per unit cost of firm k, where firm k is assumed to be the lowest cost firm in the industry. Then Figure 1 identifies the set of events S_1, where firm k is the lowest cost firm and charges C_S per unit; the set S_2, where firm k is the lowest cost firm and charges $p^*(C_L)$, the unconstrained monopoly price, per unit; and the set S_3, where firm k is not the lowest cost firm.

From the monopoly first order condition, we have $p^*(C_L) = C_L - Q^* \frac{dp^*}{dQ}$, so that $p^*(C_L) > C_L$. Furthermore, the second order condition implies $\frac{dp^*}{dC_L} > 0$, hence S_1 forms a band between the sets S_2 and S_3 as shown in Figure 1.

Figure 2 displays the same situation in terms of industry demand and minimum per unit cost C_L. Given C_L, $p^*(C_L)$ separates the diagram so that if C_S lies between C_L and $p^*(C_L)$, the output price $P = C_S$, while if $C_S > p^*(C_L)$, then $P = p^*(C_L)$.

It is convenient to characterize firms in terms of the productivity or effectiveness of R and D expenditures (as this is perceived by the various firm managers). Let α_j denote the measure of effectiveness of a dollar of R and D expenditures for firm j, where α_j is taken to be a constant independent of R_j. Then we write $f_{kj}(C_j;R_j) = f_{kj}(C_j;\alpha_j R_j)$. To keep the notation as simple as possible, we have not indicated that α_j is that perceived by firm k, but this is implied by $\alpha_j R_j$ appearing in the function $f_{kj}(\cdot)$.

Expected profits for firm k, E_k, may then be written as

$$E_k = \frac{1}{1+r} \int_0^\infty \Big[\int_{C_L}^{p^*(C_L)} Q(C_S)(C_S-C_L) \sum_{i\neq k} f_{ki}(C_S;\alpha_i R_i) \prod_{j\neq i\neq k} (1-F_{kj}(C_S;\alpha_j R_j))dC_S$$

$$+ \int_{p^*(C_L)}^{\infty} Q(p^*(C_L))(p^*(C_L)-C_L) \sum_{i\neq k} f_{ki}(C_S;\alpha_i R_i) \prod_{j\neq i\neq k} (1-F_{kj}(C_S;\alpha_j R_j)dC_S\Big] \,.$$

$$f_{kk}(C_L;\alpha_k R_k)dC_L \; - R_k \,,$$

where r is the one period interest rate.[2]

In the expression for E_k, the term $\sum_{i\neq k} f_{ki}(C_S;\alpha_i R_i) \prod_{j\neq i\neq k} (1-F_{kj}(C_S;\alpha_j R_j))$ is the pdf such that the lowest per unit cost among firms in the industry other than k is C_S. Then the first integral inside the square brackets is the expected profits for firm k, given that firm k is the low cost firm with per unit cost

2 This formulation assumes that the industry demand function approaches the price axis asymptotically. If the demand curve reaches the price axis at p such that $Q(p) = 0$, then the upper limit of integration becomes $Q^{-1}(0)$ instead of ∞.

C_L, $C_L < C_S < p^*(C_L)$, so that firm k charges C_S per unit. The second integral gives expected profits for firm k, given that firm k is the low cost firm with per unit cost C_L, $C_L < p^*(C_L) < C_S$, so that firm k charges the monopoly price $p^*(C_L)$. Weighting the sum of these integrals by $f_{kk}(C_L;\alpha_k R_k)$, the pdf that firm k's cost is C_L, and integrating, we obtain expected net profits next period for firm k. Discounting this to the present and deducting R_k then gives the discounted present value of expected profits for firm k.

III. A ''QUASISOLIPSISTIC'' INTERPRETATION OF THE R and D MODEL

In our model of firm decision making with respect to R and D, each of the n firms in the industry is simultaneously attempting to choose an optimal level of its own R and D expenditures, which depends on the R and D expenditures chosen by all other firms, without knowing what the probability beliefs of other firms are, much less their choices of R and D expenditures. One way in which to handle this problem is as follows.

Assume that each firm thinks of all other firms as possessing the same information and the same beliefs that it has, i.e., firms are ''quasi-solipsistic.'' Thus firm k ($k = 1, \ldots, n$) acts as though every firm has the same subjective probability density that firm k has as well as the same value of α. This means that firm k acts on the belief that every firm in the industry will choose the same level of R and D expenditures that firm k does in maximizing discounted expected profits. Thus firm k evaluates its first order condition for profit

maximization at the point $R_i = R_k$, $i = 1, \ldots, n$, with $\alpha_i = \alpha_k$ $i = 1, \ldots, n$ as well.

Then firm k chooses R_k to satisfy the first order conditions

$$R_k \frac{\partial E_k}{\partial R_k} = 0,\ R_k \geq 0,\ \frac{\partial E_k}{\partial R_k} \leq 0, \text{ where}$$

$$\frac{\partial E_k}{\partial R_k} = \frac{\alpha_k}{1+r} \int_0^\infty \Big[\int_{C_L}^{p^*(C_L)} Q(C_S)(C_S - C_L) \sum_{i \neq k} f_{ki}(C_S;\alpha_k R_k) \cdot$$

$$\prod_{j \neq i \neq k} (1 - F_{kj}(C_S;\alpha_k R_k))dC_S +$$

$$\int_{p^*(C_L)}^\infty Q(p^*(C_L))(p^*(C_L) - C_L) \sum_{i \neq k} f_{ki}(C_S;\alpha_k R_k) \prod_{j \neq i \neq k} (1 - F_{kj}(C_S;\alpha_k R_k))dC_S \Big]$$

$$\frac{\partial f_{kk}}{\partial s_k}(C_L;\alpha_k R_k)dC_L \quad -1, \text{ where } s_k = \alpha_k R_k .$$

Generally speaking, in what follows we will assume a regular interior solution ($R_k > 0$, $\frac{\partial^2 E_k}{\partial R_k^2} < 0$ at the optimum) to the maximization problem. However it should be pointed out that there exist cases (e.g., constant elasticity industry demand with an exponential pdf) where either no maximum exists or where a corner solution ($R_k = 0$) obtains.

Let $\frac{\partial E_k}{\partial R_k} = \phi^k(R_k, \alpha_k)$, so that $\phi^k(R_k, \alpha_k) = 0$, with $\phi^k_{R_k} \equiv \frac{\partial^2 E_k}{\partial R_k^2} < 0$ at a regular interior maximum. We next specialize our problem by assuming that all firms have the same pdfs f_{kj} over the costs of other firms; firms differ from one another only in their beliefs as to the effectiveness or productivity parameter α (and the level of R and D expenditures R). Thus firm k has the same pdfs as firm i, but firm k believes that all firms operate with a value α_k of the effectiveness parameter and with expenditures R_k while firm i believes that all firms operate with a value α_i of that parameter and expenditures R_i. What this means is that $\phi^k(R_k,\alpha_k) = \phi(R_k,\alpha_k)$ for k=1, ..., n; the functional forms ϕ^k are the same for all firms.

In our formulation of the R and D decision problem, each firm k chooses an optimal level of R and D expenditures under the assumption that every other firm, operating under the same beliefs and with the same value of α, will choose the same level of R and D expenditures that k does. This ''quasi-solipsistic'' assumption may not turn out to be correct, of course. Under <u>weak uncertainty</u> (consensus), since all firms do in fact have the same values of the effectiveness parameter α and the same pdfs, there will be a common solution (R, α) satisfying $R_k = R$ and $\alpha_k = \alpha$ where $\phi(R_k,\alpha_k) = \phi(R,\alpha) = 0$. Under weak uncertainty, the supposition under which firms make decisions turns out to be correct.

But with <u>strong uncertainty</u>, firms differ in their perception as to the value of the effectiveness parameter α that they assign to their own and other firms' R and D efforts. Hence <u>ex post</u>, firms have made mistakes. Presumably in a multiperiod model in which revelation

occurs through the outcomes of a sequence of R and D games, these mistakes will tend to disappear. We first examine the case of a one-shot R and D effort; later we look at the long run case in which perceptions of other firms' behavior are correct. In the context of the ''short run'' case, we are interested in R and D as between the weak and strong uncertainty cases, given that beliefs in the weak uncertainty case are centered at the mean of beliefs in the strong uncertainty case.

IV. R and D EXPENDITURES - CONSENSUS AND DIVERSITY

Given a regular interior maximum of discounted expected profits, we can write R and D expenditures for firm k as $R_k = R(\alpha_k)$ k = 1, ..., n, where $\phi(R(\alpha_k),\alpha_k) = 0$ from the first order condition.

Then, under strong uncertainty, where α varies from firm to firm, industrywide R and D expenditures are simply $R_D = \sum_{k=1}^{n} R(\alpha_k)$, where R_D refers to R and D expenditures under diversity.

On the other hand, under weak uncertainty, all firms are identical in all respects, including the value assigned to the effectiveness parameter α, with $\alpha_k = E(\alpha)$ for k = 1, ..., n, where $E(\alpha) = \frac{1}{n}\sum_{k=1}^{n}\alpha_k$. Let $\bar{\alpha} = E(\alpha)$. Then industry wide expenditures under weak uncertainty are given by $R_C = nR(\bar{\alpha})$, where R_C refers to R and D expenditures under consensus.

In comparing industrywide R and D expenditures under weak and strong uncertainty, Jensen's Inequality is of interest, namely:

$\sum_{k=1}^{n} R(\alpha_k) \geq nR(\bar{\alpha})$ for all $\alpha_1, \ldots, \alpha_n$ such that $\bar{\alpha} = \frac{1}{n}\sum_{k=1}^{n}\alpha_k$, if and only if $R(\alpha)$ is convex in α. In particular, a sufficient condition for strict convexity of $R(\alpha)$ is that $R''(\alpha) > 0$ for all α, where

$$R'(\alpha) = \frac{-\phi_\alpha}{\phi_R},\ R''(\alpha) = \frac{1}{\phi_R^3}\{2\phi_\alpha\phi_R\phi_{\alpha R} - \phi_R^2\phi_{\alpha\alpha} - \phi_\alpha^2\phi_{RR}\}.$$

Note that from the expression for $\frac{\partial E_k}{\partial R_k} \equiv \phi(R_k,\alpha_k)$, we have that $\phi(R,\alpha) = \alpha h(x) - 1 = 0$, where $x = \alpha R$, and $h(x)$ is a complicated integral involving R and α only in product form. Then we have

$$\phi_R = \alpha^2 h'$$

$$\phi_\alpha = \frac{R}{\alpha}\phi_R + \frac{\phi+1}{\alpha}$$

$$\phi_{RR} = \alpha^3 h''$$

$$\phi_{\alpha R} = \frac{2\phi_R}{\alpha} + \frac{R}{\alpha}\phi_{RR}$$

$$\phi_{\alpha\alpha} = \frac{R^2}{\alpha^2}\phi_{RR} + \frac{2R}{\alpha^2}\phi_R\ .$$

Thus it follows that $R''(\alpha) = \frac{2R}{\alpha^2} + \frac{1}{\alpha^2\phi_R^3}\ (4\phi_R^2 - \phi_{RR})$ which is generally ambiguous in sign. With $\phi_R < 0$, $R'' > 0$ if $\phi_{RR} \geq 4\phi_R^2$; if $\phi_R > 0$, then $R'' > 0$ if $4\phi_R^2 \geq \phi_{RR}$, but these conditions clearly depend on specific properties of the pdf's and the demand function for

output. This leads into the following basic proposition concerning the role of consensus (diversity) in promoting R and D activity.

Proposition 1. If $\phi_R > 0$, and if $\phi = 0$ bounds a strictly convex set from below (or if $\phi_R < 0$ and $\phi = 0$ bounds a strictly convex set from above), then $R_D \equiv \sum_{k=1}^{n} R(\alpha_k) > nR(\bar{\alpha}) \equiv R_C$. If $\phi_R > 0$ and if $\phi = 0$ bounds a strictly convex set from above (or if $\phi_R < 0$ and $\phi = 0$ bounds a strictly convex set from below), then $R_D < R_C$. We assume $\alpha_i \neq \alpha_k$ for some i, k in both cases.

Proposition 1 can be illustrated graphically as in Figure 3. Under divergent beliefs, firms #1 and #2 spend R_1 and R_2 for R and D respectively based upon the parameter values α_1 and α_2. Average industrywide R and D expenditure for this case is given by $\frac{R_D}{2} = \frac{R_1+R_2}{2}$. Under commonly shared beliefs where both firms have effectiveness parameter values of $\bar{\alpha}$, the resulting industrywide R and D expenditure would correspond to R_C as seen in Figure 3. $R_D > R_C$ as indicated in Proposition 1, with $\phi_R > 0$ and $\phi = 0$ bounding a strictly convex set from below.

We should note that spending on R and D by a firm is related to the value taken on by its effectiveness parameter α. The responsiveness of R and D spending to α is given by $R'(\alpha) = \frac{-\phi_\alpha}{\phi_R}$. Note that $\phi_\alpha = \frac{R\phi_R+1}{\alpha} \gtrless 0$ as $R\phi_R \gtrless -1$. For $R\phi_R < -1$, $\phi_\alpha < 0$ and $\phi_R < 0$ so that $R'(\alpha) < 0$—an increase in the effectiveness of R and D reduces R and D expenditures. On the other hand, for $R\phi_R > -1$, $\phi_\alpha > 0$ with

$R'(\alpha) \lessgtr 0$ as $\phi_R \lessgtr 0$.

Thus it might happen that R and D spending is higher under diversity than under consensus ($R_D > R_C$) but that this higher spending is concentrated among the less effective firms ($R'(\alpha) < 0$) so that less technical progress (as measured by minimum cost C_L) occurs under diversity than under consensus. We return to this issue below.

V. NASH EQUILIBRIUM IN THE LONG RUN

The quasi-solipsistic short run model is replaced in the long run by a full information Nash equilibrium model of the R and D process. In particular, assume that with repeated trials, each firm k discovers the pdf $f_{ii}(C_i;\alpha_i R_i)$ of firm i over its own cost for $i \neq k$, and in addition discovers the values α_i. Let $f_i(C_i;\alpha_i R_i) \equiv f_{ii}(C_i;\alpha_i R_i)$. Then, since each firm is known to be an expected profit maximizer, each firm can in principle solve for the values $R_1,\dots,R_n$ that jointly maximize expected profits for firms $1,\dots,n$, when these firms treat other firms as operating at their expected profit maximizing R and D spending levels. For example, firm k chooses R_k to

$$\max E_k = \frac{1}{1+r} \int_0^{\infty}\Big[\int_{C_L}^{P^*(C_L)} Q(C_S)(C_S-C_L) \sum_{i\neq k} f_i(C_S;\alpha_i R_i) \cdot$$

$$\prod_{j\neq i\neq k} (1-F_j(C_S;\alpha_j R_j))dC_S + \int_{p^*(C_L)}^{\infty} Q(p^*(C_L))(p^*(C_L)-C_L)$$

$$\sum_{i\neq k} f_i(C_S;\alpha_i R_i) \prod_{j\neq i\neq k} (1-F_j(C_S;\alpha_j R_j))dC_S\Big] f_k(C_L;\alpha_k R_k)dC_L \quad -R_k$$

where R_i $i \neq k$ is treated as fixed at its expected profit maximizing level, which in turn is calculated by maximizing E_i with R_j $j \neq i$ fixed at expected profit maximizing levels.

Let the term inside the square brackets above be denoted by $\gamma_k(C_L)$. Then the first order condition is given by

$$\frac{\partial E_k}{\partial R_k} = \frac{\alpha_k}{1+r}\int_0^{\infty}\gamma_k(C_L)\frac{\partial f_k}{\partial s_k}dC_L - 1 \leq 0,$$

$$R_k\frac{\partial E_k}{\partial R_k} = 0, \text{ where } s_k = \alpha_k R_k .$$

Note that $\gamma_k(C_L)$ is independent of R_k, but depends on all R_j $j \neq k$.

Then these first order conditions determine some level of industrywide R and D expenditures $R = \sum_{k=1}^{n} R_k$. It is now possible to say something as well about the rate of technical progress in the industry as well as the price that might prevail for the industry's product.

In examining the impact of a certain pattern of R and D spending on the level of technological progress, what we are concerned with, of course, is the minimum level of cost achieved. Thus, given a pattern of R and D expenditures of $(R_1, \dots, R_n)$ by firms $1, \dots, n$, let $h(C_L|R_1, \dots, R_n)$ denote the probability that the minimum per unit cost achieved is C_L. Then h can be written as:

$$h(C_L|R_1, \dots, R_n) = \sum_{i=1}^{n} \prod_{j\neq i} (1-F_j(C_L;\alpha_j R_j) \quad f_i(C_L;\alpha_i R_i)$$

Then it follows that the expected (minimum) per unit cost is given by

$$E(C_L|R_1, \ldots, R_n) = \int_0^\infty C_L \sum_{i=1}^{n} \left\{ \underset{j\neq i}{\pi} (1-F_j(C_L;\alpha_j R_j) \right\} f_i(C_L;\alpha_i R_i)\, dC_L$$

One interesting issue that remains to be explored is that of the extent to which the rents from technical progress are captured by the innovating firm, and the extent to which they are passed on to the public in the form of lower prices. This involves an investigation into the probability distribution over price P.

As in the previous sections, let C_S be the second lowest per unit cost in the industry and let C_L be the lowest per unit cost. Then if P is the price per unit of output, we have either

$$P = C_S \text{ with } C_L \leq C_S \leq p^*(C_L), \text{ or} \tag{1}$$

$$P = p^*(C_L) \text{ with } C_L \leq p^*(C_L) \leq C_S. \tag{2}$$

Figure 4 indicates the classes of events associated with these two possibilities.

Then $pdf_1(P)$ = pdf such that $P = C_S$ with $C_L \leq C_S \leq p^*(C_L)$ is given by

$$pdf_1(P) = \int_{p^{*-1}(P)}^{P} \sum_{i=1}^{n} f_i(C_L) \sum_{k\neq i} f_k(P) \underset{j\neq k\neq i}{\pi} (1-F_j(P))dC_L.$$

Similarly, $pdf_2(P)$ = pdf such that $C_L \leq p^*(C_L) \leq C_S$ is given by

$$pdf_2(P) = \int_P^\infty \sum_{i=1}^{n} f_i(p^{*-1}(P)) \sum_{k\neq i} f_k(C_S) \underset{j\neq k\neq i}{\pi} (1-F_j(C_S))dC_S$$

with $EP = \int_0^\infty P\{pdf_1(P) + pdf_2(P)\}\, dP.$[3]

In principle, a comparison could be made between diversity and consensus in the long run case just as in the short run case. However, things are more complicated here in that the system of n simultaneous first order conditions would have to be solved to obtain the configuration of R and D expenditures $R_1,\ldots,R_n$ for the diversity case. It is clear that except in special cases, characterizing the Nash equilibrium under diversity is an interesting feat. Thus we turn at this point to a special case which offers some hope of tractability in solving — the case of exponential pdf's.

VI. A SPECIAL CASE: EXPONENTIAL PROBABILITY DISTRIBUTIONS

Consider the case of an exponential pdf, a case investigated in a different context by Lowry (2) and Lee and Wilde (3). For the exponential case we have

$$F(C;\alpha R) = 1 - e^{-\alpha RC} \text{ with}$$
$$f(C;\alpha R) = \alpha Re^{-\alpha RC} \quad 0 \leq C \leq \infty .$$

Note that $\frac{\partial F}{\partial R} = C\alpha e^{-\alpha RC} > 0$, which means that an increase in R and D spending leads to an unambiguously ''better'' distribution of per unit cost, since a higher level of R results in a distribution of per unit cost that stochastically dominates the distribution associated with any lower level of R.

3 If the marginal revenue function crosses the Q axis at Q so that $p^*(0) > 0$, then the lower limit of integration in EP for pdf_2 is $p^*(0)$.

With exponential pdf's, the first order conditions in the short run case can be written as $\emptyset(R_k, \alpha_k) =$

$$\frac{(n-1)}{1+r}\alpha_k^2 R_k \int_0^\infty \Big[\Big[\int_{C_L}^{p^*(C_L)} Q(C_S)(C_S-C_L)e^{-(n-1)\alpha_k R_k C_S} dC_S$$

$$\int_{p^*(C_L)}^\infty Q(p^*(C_L))(p^*(C_L) - C_L)e^{-(n-1)\alpha_k R_k C_S} dC_S\Big](1-\alpha_k R_k C_L)e^{-\alpha_k R_k C_L} dC_L$$

$$-1 = 0 \quad k = 1,\ldots,n, \text{ for } R_k > 0 .$$

In the case of the exponential pdf, we can write the expression for expected minimum cost as

$$E(C_L | R_1, \ldots, R_n) = \int_0^\infty C_L \sum_{i=1}^n \alpha_i R_i e^{-\sum_{j=1}^n \alpha_j R_j C_L} dC_L$$

hence

$$E(C_L | R_1, \ldots, R_n) = \frac{1}{\sum_{j=1}^n \alpha_j R_j} .$$

Thus, it turns out that in evaluating the impact on per unit cost of a certain level of R and D spending, we must take into account the effectiveness of that spending as well as its amount. The short run comparison here is between $\sum_{k=1}^n \alpha_k R_k$ under diversity and $n\bar{\alpha}R(\bar{\alpha})$ under consensus. Again, by Jensen's inequality ''diversity pays'' if $\psi(\alpha) \equiv \alpha R(\alpha)$ is convex in α.

A sufficient condition for strict convexity of $\alpha R(\alpha)$ is in turn provided by $\psi''(\alpha) > 0$ for all α, where

$$\psi''(\alpha) = 2R'(\alpha) + \alpha R''(\alpha), \text{ where}$$

$$R'(\alpha) = -\emptyset_\alpha/\emptyset_R$$

$$R''(\alpha) = \frac{1}{\emptyset_R^3}\left[2\emptyset_\alpha\emptyset_R\emptyset_{\alpha R} - \emptyset_R^2\emptyset_{\alpha\alpha} - \emptyset_\alpha^2\emptyset_{RR}\right] = \frac{2R}{\alpha^2} + \frac{1}{\alpha^2\emptyset_R^3}(4\emptyset_R^2 - \emptyset_{RR}).$$

Hence $\psi''(\alpha) = \dfrac{2\emptyset_R^2-\emptyset_{RR}}{\alpha\emptyset_R}$. Thus we have the following result.

<u>Proposition 2</u>. Given exponential pdf's over the payoffs from R and D, then if $\emptyset_R < 0$ and $\emptyset_{RR} \geq 4\emptyset_R^2$, R and D spending in the short run is greater under diversity than under consensus, and there is more technical progress (as measured by EC_L) under diversity than under consensus as well. The same is true if $\emptyset_R > 0$ with $\emptyset_{RR} \leq 2\emptyset_R^2$. However with $\emptyset_R < 0$, $4\emptyset_R^2 \geq \emptyset_{RR} \geq 2\emptyset_R^2$, EC_L is lower under diversity than consensus, but spending is higher under consensus than under diversity. Similarly, $\emptyset_R > 0$, $2\emptyset_R^2 \leq \emptyset_{RR} \leq 4\emptyset_R^2$ implies higher spending under diversity but lower EC_L under consensus.

Similarly, with exponential pdf's, the expression for EP, expected price, can be written as

$$EP = \sum_{i=1}^n \int_0^\infty P \sum_{k=1}^n \alpha_k R_k e^{-(\sum_{j\neq i} \alpha_j R_j P + \alpha_i R_i p^{*-1}(P))}$$

$$- \sum_{j\neq i} \alpha_j R_j e^{-\sum_{j=1}^{n} \alpha_j R_j P} \quad dP,$$

which can be simplified to

$$EP = \int_0^\infty Pn\psi(\bar{\alpha})e^{-n\psi(\bar{\alpha})P}\Big[\sum_{i=1}^{n} e^{-\psi(\alpha_i)(p^{*-1}(P)-P)} - (n-1)\Big]dP$$

where $\psi(\bar{\alpha}) = \bar{\alpha}R(\bar{\alpha})$, $\psi(\alpha_i) = \alpha_i R_i$.

Let EP_D be expected price under diversity and EP_C be expected price under consensus, with $\Theta(\alpha)$ defined by $\Theta(\alpha) = e^{-\ (\alpha)(p^{*-1}(P)-P)}$. By Jensen's Inequality, $EP_D > EP_C$ for all non-degenerate frequency distributions over α if $\Theta''(\alpha) > 0$.

We have

$$\Theta'(\alpha) = -\psi'(\alpha)(p^{*-1}(P)-P)e^{-\psi(\alpha)(p^{*-1}(P)-P)}$$

with

$$\Theta''(\alpha) = \Big[\psi'(\alpha)(p^{*-1}(P)-P)\Big]^2 e^{-\psi(\alpha)(p^{*-1}(P)-P)}$$

$$-\psi''(\alpha)(p^{*-1}(P)-P)e^{-\psi(\alpha)(p^{*-1}(P)-P)}$$

Then $p^{*-1}(P) < P$ by the assumption of downward sloping industry demand. Thus a sufficient condition for $\Theta''(\alpha) > 0$ is $\psi''(\alpha) > 0$. This leads into the following.

Proposition 3. Given exponential pdf's over the payoffs from R and D expenditures, expected price under diversity of beliefs is greater than expected price under consensus, when EC_L is lower under diversity.

Thus we arrive at the paradoxical conclusion that with exponential pdf's over the payoffs from R and D expenditures and with expected minimum cost less under diversity of beliefs than under consensus, expected price is higher under diversity than under consensus. Because the industry operates with positive R and D expenditures only in the elastic range of the demand curve, this means that under these conditions expected profits (before deducting R and D expenditures) are higher under diversity than under consensus. Intuitively, what is going on in this market is that under diversity, there is a larger variance of per unit cost among the firms in the industry than there is under consensus, due to the variation in values of α among firms. This larger variance of per unit cost under diversity acts to increase the probability that the lowest cost firm will be able to charge the unconstrained monopoly price, or, more generally, acts to lessen the constraint imposed on price and profits by the latent competition of the second lowest cost producer. Because of the special properties of exponential densities, we have no assurance, however, that this result holds in general. It still has not been established that the paradox of lower expected minimum cost and higher expected price under diversity holds under pdf's other than the exponential.

VII. OPTIMALITY AND MARKET DETERMINED R AND D EXPENDITURES

As the preceding analysis makes clear, the positive economics of R and D expenditures in a market environment leads to a minimum of predictive statements. Perhaps turning to normative economics might help. Here we have the two polar cases of Arrow's argument (1) that lack of appropriability leads to less than optimal amounts of R and D expenditures, and Hirshleifer's (2) that ''distributive gains'' associated with speculation can lead to a more than optimal level of R and D spending. Actually, in the model of the present paper, appropriability is not a problem, since the low cost firm is assumed to wield monopoly power (subject to threats from the second low cost firm) in its output market. Moreover, we have in effect assumed away the possibilities of speculative opportunities as well. What remains to cause problems from the point of view of optimality are (a) monopoly power by the low cost firm once its R and D is successful; and (b) problems related to duplication of efforts by firms competing for the chance to become the low cost firm.

We do not attempt to address the fundamental conceptual problems that plague the notion of optimality under uncertainty, but instead take the easy way out and use as our optimality criterion expected (discounted) consumer's surplus, using the pdf's $f_i(C_i;\alpha_i R_i) \equiv f_{ii}(C_i;\alpha_i R_i)$ $i = 1,\ldots,n$ as ''objective'' pdf's.

Thus, let W denote welfare. W is given by

$$W = -\sum_{i=1}^{n} R_i - \frac{1}{1+r}\int_0^\infty \left\{\int_{C_L}^\infty Q(p)dp\right\} h(C_L|R_1,\ldots,R_n)dC_L \ ,$$

where, as above, $C_L = \min_i C_i$, and

$$h(C_L|R_1,\ldots,R_n) = \sum_{i=1}^{n} f_i(C_L;\alpha_i R_i) \prod_{j\neq i} (1-F_j(C_L;\alpha_j R_j)) \ .$$

It follows that at an optimum, where W is maximized, the following first order conditions are satisfied:

$$\frac{\partial W}{\partial R_k}R_k = 0, \quad R_k \geq 0, \quad \frac{\partial W}{\partial R_k} \leq 0 \quad \text{where}$$

$$\frac{\partial W}{\partial R_k} = -1 - \frac{1}{1+r}\int_0^\infty \left\{\int_{C_L}^\infty Q(p)dp\right\} \frac{\partial h}{\partial R_k}dC_L, \text{ with}$$

$$\frac{\partial h}{\partial R_k} = \alpha_k\left\{\frac{\partial f_k}{\partial s_k}\prod_{j\neq k}(1-F_j) - \frac{\partial F_k}{\partial s_k}\sum_{i=1}^{n} f_i \prod_{j\neq k\neq i}(1-F_j)\right\}, \text{ where}$$

$s_k = \alpha_k R_k$.

In particular, for the case of exponential pdf's,

$$h(C_L|R_1,\ldots,R_n) = \sum_{i=1}^{n}\alpha_i R_i e^{-\sum_{i=1}^{n}\alpha_i R_i C} \quad \text{so that}$$

$$\frac{\partial h}{\partial R_k} = \alpha_k(1-C_L\sum_{i=1}^{n}\alpha_i R_i)e^{-\sum_{i=1}^{n}\alpha_i R_i C_L} . \quad \text{Let}$$

$$\gamma(C_L;R_1,\ldots,R_n) = (1-C_L\sum_{i=1}^{n}\alpha_i R_i)e^{-\sum \alpha_i R_i C_L} \quad \text{so that}$$

$\frac{\partial h}{\partial R_k} = \alpha_k\gamma(c_L;R_1,\ldots,R_n)$, where γ is the same for $k = 1,\ldots,n$.

Then from the first order condition we have

$$\frac{-\alpha_k}{1+r}\left[\int_0^\infty \left\{\int_{C_L}^\infty Q(p)dp\right\}\gamma(C_L;R_1,\ldots,R_n)dC_L\right] \leq 1$$

for $k = 1,\ldots,n$, where the term inside the square brackets is independent of k.

Thus it follows that for $\alpha_k = \max_i \alpha_i$, $R_k > 0$ with the first order condition holding as an equality. If $\alpha_j < \max_i \alpha_i$, then $\frac{\partial W}{\partial R_j} < 0$ and $R_j = 0$. Hence we arrive at the basic nondiversification result for the exponential case.

Proposition 4. Given that all pdf's are exponential, then an optimal allocation of resources to R and D expenditures involves positive expenditures only by that firm k for which $\alpha_k = \max_i \alpha_i$.

In contrast, consider the first order conditions characterizing a full information long-run Nash equilibrium of market determined R and D expenditures, assuming exponential pdf's. These conditions are given by

$$\frac{\alpha_k}{1+r}\int_0^\infty V(C_L;a_k)(1-\alpha_k R_k C_L)e^{-\alpha_k R_k C_L}dC_L \leq 1$$

$k = 1,\ldots,n$, where

$$V(C_L;a_k) = \int_{C_L}^{p^*(C_L)} Q(C_S)(C_S-C_L)a_k e^{-a_k C_S}dC_S$$

$$+ \int_{p^*(C_L)}^{\infty} Q(p^*(C_L))(p^*(C_L))a_k e^{-a_k C_S}$$

is the expected value of revenue to firm k given that firm k has a per unit cost of C_L, where $a_k = \sum_{i\neq k}\alpha_i R_i$.

Let $t(C_S;a_k) = a_k e^{-a_k C_S}$. Then $T(C_s;a_k) = \int_0^{C_S} t(x;a_k)dx$ is given by $T(C_S;a_k) = 1 - e^{-a_k C_S}$ with $\frac{\partial T}{\partial a_k} = C_S e^{-a_k C_S} > 0$.

Hence $t(C_S;a_k)$ is stochastically dominated by $t(C_S;a_k')$ for $a_k > a_k'$. Thus it follows by the fundamental stochastic dominance theorem (see Quirk and Saposnik (6)) that $V(C_L;a_k') > V(C_L;a_k)$ for $a_k > a_k'$. That is, an increase in $\sum_{i\neq k}\alpha_i R_i$ lowers the conditional expected revenue of firm k, for any value of C_L.

Rewrite the first order condition as

$$\frac{\alpha_k}{1+r}\int_0^\infty V(C_L;a_k)\gamma(C_L;\alpha_k R_k)dC_L \leq 1, \text{ where}$$

$$\gamma(C_L;\alpha_k R_k) = (1-\alpha_k R_k C_L)e^{-\alpha_k R_k C_L}. \text{ Then}$$

$$\Gamma(C_L;\alpha_k R_k) = \int_0^{C_L}(1-\alpha_k R_k x)e^{-\alpha_k R_k x}dx = C_L e^{-\alpha_k R_k C_L}$$

$$\text{with } \frac{\partial \Gamma}{\partial s_k} = -C_L^2 e^{-s_k C_L} < 0 \quad (s_k = \alpha_k R_k). \text{ Hence}$$

$\gamma(C_L;s_k)$ is stochastically dominated by $\gamma(C_L;s_k')$ for $s_k' > s_k$, and

$$\int_0^\infty V(C_L;a_k)\gamma(C_L;s_k')dC_L > \int_0^\infty V(C_L;a_k)\gamma(C_L;s_k)dC_L$$

for $s_k' > s_k$, which in turn implies

$$\int_0^\infty V(C_L;a_k')\gamma(C_L;s_k')dC_L > \int_0^\infty V(C_L;a_k)\gamma(C_L;s_k)dC_L$$

for $a_k' < a_k$ and $s_k' > s_k$.

It immediately follows from this that at a Nash equilibrium, $R_k > 0$ only for that k for which $\alpha_k = \max_i \alpha_i$, since, with $\alpha_k R_k > \alpha_i R_i$ for all i, and with $\frac{\alpha_k}{1+r}\int_0^\infty V(C_L;a_k)\gamma(C_L;s_k)dC_L = 1$, then

$\frac{a_i}{1+r}\int_0^\infty V(C_L;a_i)\gamma(C_L;s_i)\,dC_L < 1$ for $i \neq k$.

Hence under a market determined Nash equilibrium, as at an optimum, there are positive R and D expenditures only by the most productive firm. And since there is only one firm in the market, that firm acts as a monopolist, choosing R_k such that the following is satisfied,

$$\frac{\alpha_k}{1+r}\int_0^\infty Q(p^*(C_L))(p^*(C_L)-C_L)(1-\alpha_k R_k C_L)e^{-\alpha_k R_k C_L} = 1.$$

At an optimum, we have

$$\frac{\alpha_k}{1+r}\int_0^\infty \int_{C_L}^\infty Q(p)\,dp \quad (1-\alpha_k R_k C_L)e^{-\alpha_k R_k C_L} = 1,$$

hence the usual monopoly distortion occurs.

Proposition 5. Given that all pdf's are exponential, then a market determined allocation of R and D expenditures involves positive expenditures only by that firm k for which $\alpha_k = \max_i \alpha_i$, and the amount spent is less than that spent at an optimum. This leads to less technical progress (higher EC_L) and a higher expected price than at an optimum.

This analysis raises the general question of how restrictive must be the assumptions on the pdf's f_i in order to obtain a nondiversification result with respect to R and D expenditures in the full information Nash equilibrium case. Proposition 6 gives a sufficient condition for nondiversification, one which is satisfied for the case of uniform pdf's, and one which has some intuitive

appeal. It is not known the extent to which this sufficient condition can be weakened.

Proposition 6. Assume that costs C_i are independently distributed, all pdf's f_i having the same functional form and differing only in the values assigned to the parameter $s_i = \alpha_i R_i$. Thus $f_i(C_i;s_i) = f(C_i;s_i)$. As above, let $F(C;s_i) = \int_0^C f(C_i;s_i)\,dC_i$, with $\frac{\partial F(C;s_i)}{\partial s_i} > 0$ for all $C > 0$. Further, assume that for $C > 0$, $\frac{\partial F(C;s_i)}{\partial s_i} > \frac{\partial F(C;s_j)}{\partial s_j}$ for $s_i > s_j$, i.e., F is strictly convex in s. Then an optimal allocation of R and D expenditures is one in which there is a positive amount of expenditures only for that firm k for which $\alpha_k = \max_i \alpha_i$.

Proof:

Let $\Phi(C) = \int_0^C \phi(x;R_1,\dots,R_n)\,dx$, where $\phi(x;R_1,\dots,R_n) = \sum_{i=1}^n f(x;s_i) \prod_{j\neq i}(1-F(x;s_j))$. Consider the problem of maximizing $\Phi(C)$ subject to $\sum_{i=1}^n R_i = \bar{R}$, where $\bar{R}$ is a fixed constant. We will show that under the conditions of the proposition, the solution to this problem involves choosing $R_k = \bar{R}$ for k such that $\alpha_k = \max_i \alpha_i$.

The first order conditions are

$$\frac{\partial\Phi}{\partial R_i} - \lambda \leq 0 \qquad i = 1,\dots,n$$

where

$$\frac{\partial\Phi}{\partial R_i} = \alpha_i\int_0^C\Big[-\sum_{j\neq i} f(x;s_j)\underset{k\neq j\neq i}{\pi}(1-F(x;s_k))\frac{\partial F(x;s_i)}{\partial s_i}\Big]dx$$

$$+\ \alpha_i\int_0^C \frac{\partial f(x;s_i)}{\partial s_i}\underset{j\neq i}{\pi}(1-F(x;s_j))dx\ .$$

The second integral may be integrated by parts to obtain

$$\int_0^C \frac{\partial f(x;s_i)}{\partial s_i}\underset{j\neq i}{\pi}(1-F(x;s_j))dx = \frac{\partial F(C;s_i)}{\partial s_i}\underset{j\neq i}{\pi}(1-F(C;s_j))$$

$$+\int_0^C \frac{\partial F(C;s_i)}{\partial s_i}\sum_{j\neq i} f(x;s_j)\underset{k\neq j\neq i}{\pi}(1-F(x;s_k))dx$$

Hence the first order conditions become

$$\alpha_i\frac{\partial F(C;s_i)}{\partial s_i}\underset{j\neq i}{\pi}(1-F(C;s_j)) \leq \lambda \qquad i = 1,\dots,n$$

Now with $\alpha_k = \max\limits_i \alpha_i$, the allocation $R_k = \overline{R}, R_i = 0 \quad i \neq k$ satisfies these conditions with $\lambda = \alpha_k\frac{\partial F(C;\alpha_k\overline{R})}{\partial s_k} > \alpha_i\frac{\partial F(C;0)}{\partial s_i}(1-F(C;\alpha_k\overline{R}))$ for $i\neq k$. Strict convexity of F in s guarantees that this allocation is the maximizer of $\Phi(C)$ subject to $\sum_{i=1}^{n} R_i = \overline{R}$. To see this, assume $R_k > 0$ and $R_i > 0$ for some $i \neq k$. Then if $dR_k = -dR_i > 0$, we have

$$d\Phi = dR_k\Big[\alpha_k\frac{\partial F(C;s_k)}{\partial s_k}(1-F(C;s_i)) - \alpha_i\frac{\partial F(C;s_i)}{\partial s_i}(1-F(C;s_k))\Big]\underset{j\neq i\neq k}{\pi}(1-F(C;s_j)).$$

For $\alpha_k > \alpha_i$, $s_k > s_k$, $d\Phi > 0$, hence an optimum occurs at $R_k = \overline{R}$. Consider next the optimizing problem

$$\max -\sum_{i=1}^{n} R_i - \int_0^\infty\Big\{\int_C^\infty P(x)\,dx\Big\}\phi(C;R_1,\dots,R_n)\,dC$$

Then since $\int_C^\infty P(x)dx$ is a monotone decreasing function of C, and since $\phi(C;0,\dots,\overline{R},0,\dots,0)$ ($\overline{R}$ in the k^{th} position) is stochastically dominated by every other feasible allocation, it follows that an optimal allocation of R and D spending involves positive spending only by firm k.

VII. SUMMARY

The problem we have investigated is that of the role of diverse beliefs so far as R and D activities in an industry are concerned. Using a simplified short run ''quasi-solipsistic'' model of the R and D process, we have shown that the effect of diversity on spending, on the rate of technical progress, and on expected price is highly sensitive to the properties of the pdf's over the payoffs from R and D, and of the demand function. Under a long run full information Nash equilibrium, and with exponential pdf's, all R and D spending is concentrated in the most productive firm, which spends less than an optimal amount, leading to a lower rate of technical progress and to a higher expected product price than at the optimum.

REFERENCES

1. Arrow, K. J. ''Economic Welfare and the Allocation of Resources for Invention,'' in <u>the Rate and Direction of Economics Activity</u>, NBER, Princeton, 1962.

2. Hirshleifer, J. ''The Private and Social Value of Information and the Reward to Inventive Activity.'' <u>American Economic Review</u>, 1971.

3. Klein, Burton. <u>Dynamic Economics</u>. Cambridge, MA: Harvard University Press, 1977.

4. Loury, Glenn C. ''Market Structure and Innovation.'' <u>Quarterly Journal of Economics</u>, 1979.

5. Lee, Tom and Louis Wilde. ''Market Structure and Innovation: A Reformulation.'' <u>Quarterly Journal of Economics</u>, 1980.

6. Quirk, J. and R. Saposnik. ''Admissibility and Measurable Utility Functions.'' <u>Review of Economic Studies</u>, 1962.

7. Savage, L. J. <u>The Foundation of Statistics</u>. New York: Wiley, 1954.

Figure 1. C_L and C_S

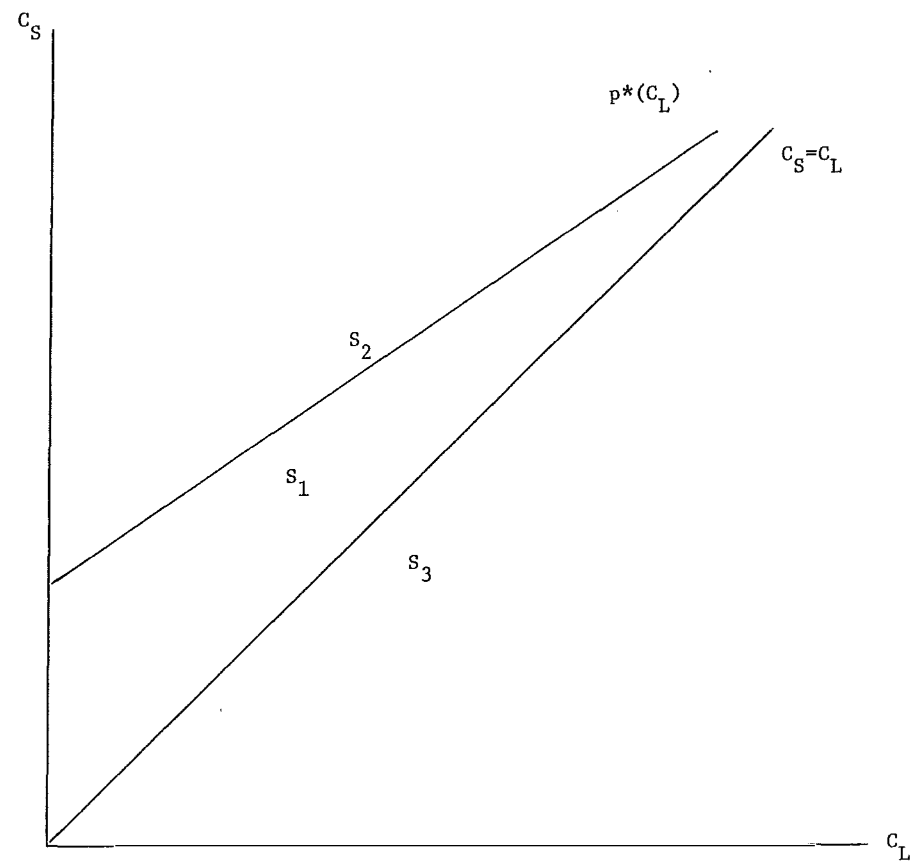

Figure 2. Industry Demand and C_L

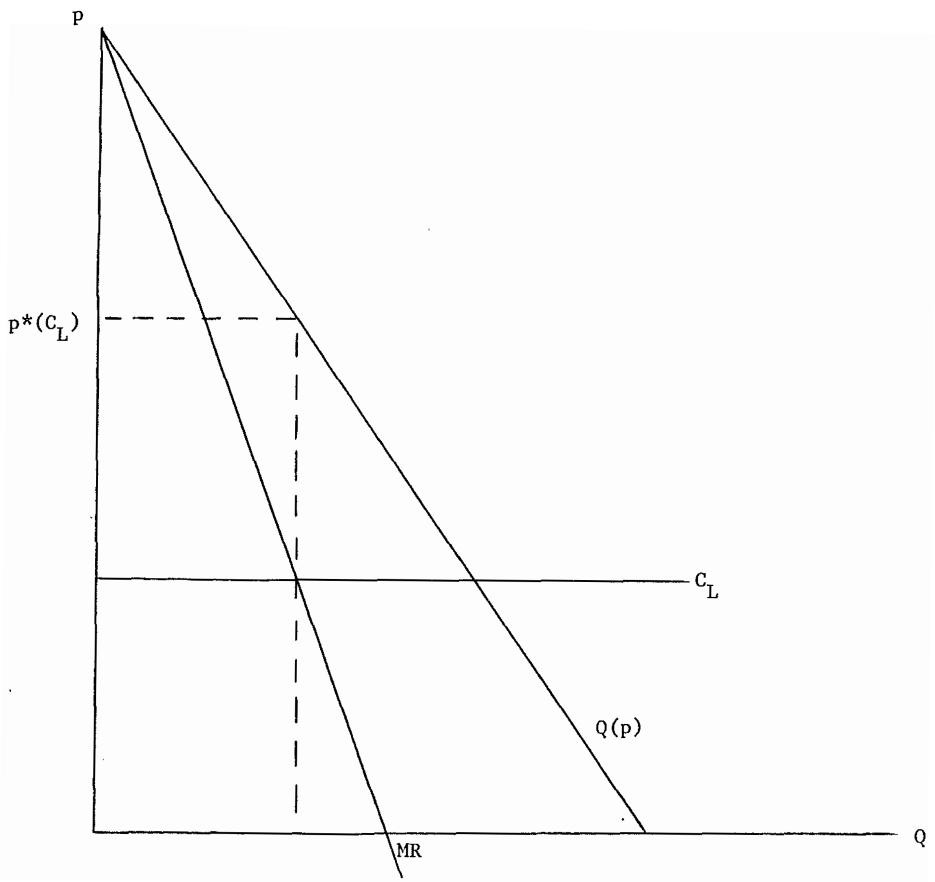

Figure 3. R_C and R_D

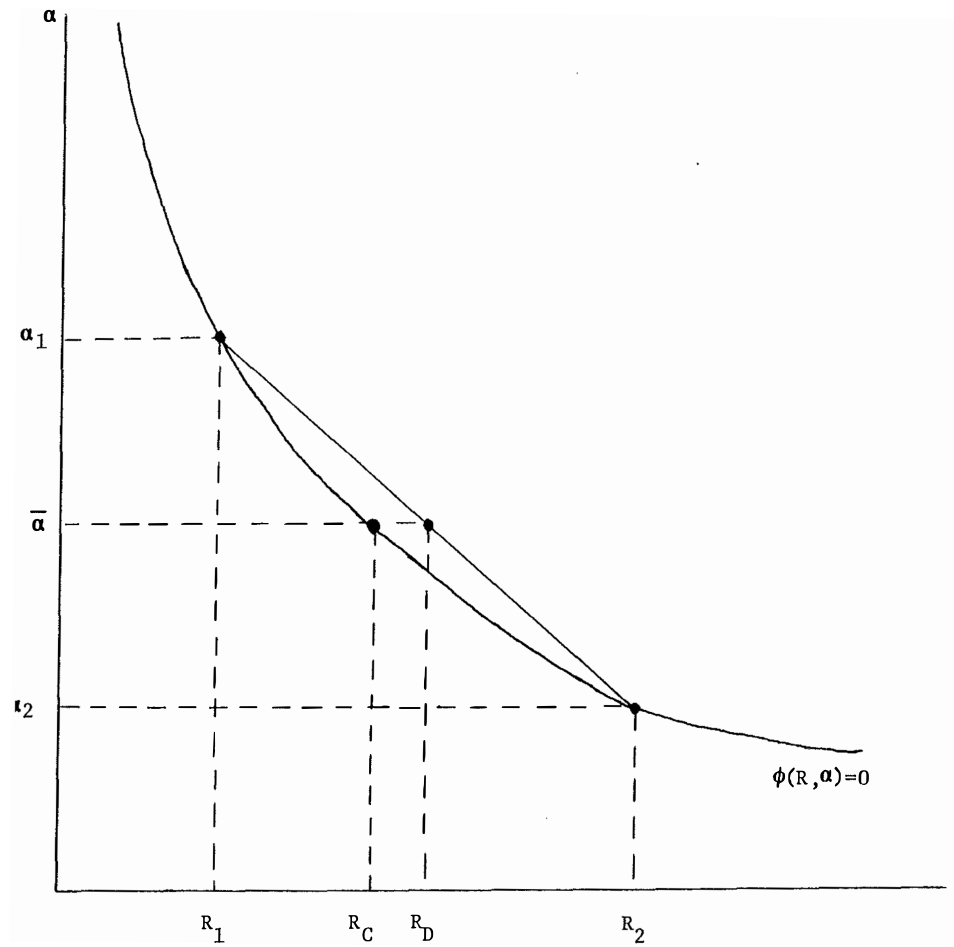

Figure 4. Deriving the pdf over P